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Computational Non-Newtonian Hemodynamics of Small Vessels

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Abstract

The significance of non-Newtonian hemodynamics of small blood vessels is addressed via the description and critical discussion of cogent models within Computational Fluid Dynamics (CFD) software, in this case ANSYS-CFX. Several applicable hemodynamical shear-thinning models are presented and the relevance with respect to prediction of Shear Strain Rate (SSR) rigorously examined, in order to critically evaluate salient literature. It is found that the small vessels explored, in line with the aforementioned literature, that non-Newtonian evaluation of the fluid behavior is indeed negligible. The work presented herein is a precursor to investigation of more complex geometries and hemodynamic simulations, which are being actively researched. This is a technical note which attempts to address the significance of Newtonian/non-Newtonian flows in small blood vessels.

Keywords: Hemodynamics; Fluid Dynamics; Shear Stress Rate; Software; Viscosity; Thrombosis

Introduction

With the advancement of commercial Computational Fluid Dynamics (CFD) software [1], the use of non-Newtonian hemodynamic modelling is becoming more manageable. An excellent treatise has been provided by Steinman [2] with respect to the underlying assumptions of blood flow modelling. The text [2] initially performs critical analyses of potential impacts on computed hemodynamics and outlined modelling assumptions with respect to the constitutive properties of blood flow and vessel walls. Here, strong analogies between an AC electrical (pulsating) circuit and the cardiovascular system are presented, with the blood viscosity providing the resistive component and the compliant walls the capacitance. In such cases:

Blood may be modelled as a Newtonian fluid, perhaps with adjustments to the apparent viscosity based on the vessel sizes included in the circuit. Steinman, 2012 [2].

Prima-facia this appears to be a rather bold statement, especially since much of literature, some of which is detailed in another conical text [3], maintains that blood is a complex non-Newtonian fluid and therefore should be treated as such in salient computational models. However, we have recently shown [5] that for small arteries the use of non-Newtonian models, in such software [1], has little affect on calculations of Shear Strain Rate (SSR), which in turn are related to thrombosis formation [4]. The inclusion of non-Newtonian behaviour is becoming increasingly pertinent as larger parts of the cardiovascular network become subject of CFD modelling; so is directly related to computational expense. This note therefore essentially critically evaluates Professor Stienman's statement with particular attention being directed toward CFD modelling of smaller arteries with an outer radius (R_o) of 1:25mm [5]. These values were chosen as they represent the vessels used in many microvascular surgical operations, such as free-tissue transfer for post oncological and post traumatic reconstruction.

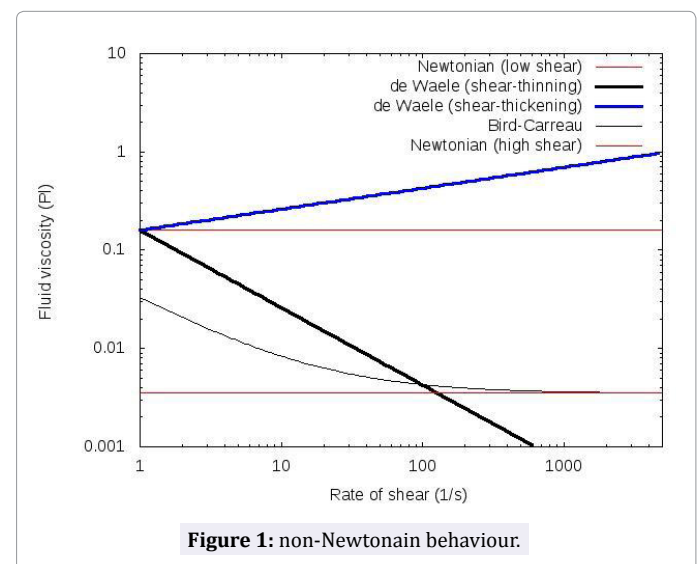
Appropriate non-Newtonian Models

The most popular model of non-Newtonian behaviour was originally presented in what is now regarded as landmark publication by de Waele [6]. Here a formal mathematical, albeit empirical, definition of a non-Newtonian fluid is offered for the first time. The model being essentially an extension of the Newtonian case relating the shear stress (τ) to the shear strain rate ($\dot{\gamma}$), thus:

$$\tau = K\dot{\gamma}^n = (K\dot{\gamma}^{n-1})\dot{\gamma}$$

with the terms in the parenthesis on the far right of this expression being termed the apparent viscosity. Despite the model's obvious lack of dimensionality, it is true to say that it is still most popular, probably due to its simplicity. Here the index (n) can be thought of a measure of divergence from Newtonian ($n = 1$) behaviour. This model allows for the description of two other types of non-Newtonian fluids: those that increase in viscosity with increasing shear rate (shear-thickening, $n > 1$) and conversely the so-called shear-thinning fluids ($n < 1$). It being generally accepted in the literature [2,3] that blood is shear-thinning, only values of an index less than unity need be considered. The specifics of these behaviors are shown in figure 1, where a de Waele blood-like shear-thinning fluid is modelled, i.e. $n = 0.2128$ [9]. Here a 160 fold reduction in apparent viscosity is evident from a low-shear viscosity value of 0.16Pl ($= \kappa$) over a SSR range of 600 s^{-1} . Also included in figure 1 for comparative purposes is an empirically validated high shear viscosity value of $3.2\text{ }\mu\text{Pl}$ for a de Weale shear-thickening fluid (analogous to Polyethylene glycol) and a further non-Newtonian (Bird-Carreau) fluid as detailed in proceeding sub-section.

The inclusion of the Bird-Carreau model in figure 1 also clearly illustrates one of the major drawbacks with the de Waele model. In that it is not bounded by either of the low or high shear limits indicated, unlike other shear-thinning models available in



commercial CFD codes [1], such as those of Cross, Bird-Carreau and Carreau-Yasuda. [7].

Cross Model

This is the simplest of the three most popular non-Newtonian models completely dedicated to shear-thinning behavior. Here the apparent viscosity is described by:

$$\mu(\gamma) = \frac{\mu_o}{1 + (\lambda\gamma)^n}$$

where λ is a time constant, ensuring that the model is dimensionally consistent. It allows for constant viscosity (μ_o) at low

strain-rates but is unrealistic at high strain-rates as the viscosity tends to zero. This said, it can be used quite successfully in describing the flow characteristics of a number pseudoplastics such as dilute macromolecular (e.g. carbohydrates and proteins) and polymeric solutions that are in some ways analogous to blood.

Carreau Models

Both the Bird-Carreau and Carreau-Yasuda models solve the shortcomings of their predecessors by allowing for limiting behaviour in the viscosity at higher strain-rates; a phenomenon which is much more cogent to cardiovascular flows [2]. The Bird-Carreau model is essentially an extension of the cross model:

$$\mu(\gamma) = \frac{\mu_o - \mu_\infty}{1 + (\lambda\gamma)^2} + \mu_\infty$$

here the high shear viscosity is μ_∞ , and represents the independence of the viscosity in the fluid at high strain rates. A more generalized form is obtained by replacing the powers of 2 and 1/2 with respectively a and $1/a$, to form of the so-called Carreau-Yasuda model thus:

$$\mu(\gamma) = \frac{\mu_o - \mu_\infty}{1 + (\lambda\gamma)^a} + \mu_\infty$$

where a is the Yasuda exponent. It is worth noting that both of these models revert to the Newtonian case, where the stress is directly proportional to the strain-rate, when the time constant is zero (i.e. the viscosity is time-invariant, $\lambda = 0$) and ipso-facto a de Waele power index of unity ($n = 1$).

The viscosity predictions from the pseudoplastic blood models described in this section are shown as a function of strain-rate in figure 2. Here identical blood parameters as per the reference [9] where used in all computations namely: $\mu_o = 0.16\text{Pl}$, $\mu_\infty = 3.5\text{Pl}$, λ

$= 8.2\text{s}$, $a = 0.64$ and $n = 0.2128$. As expected the closest agreement is shown between two, the Carreau models with the cross model appearing to be less relevant especially at high strain rates. Also included are averaged digitized data obtained from rather clever Lattice Boltzmann Method (LBM) employed by Boyd, et al. [9,10], these data showing reasonable agreement with Carreau models. Also of note that the fact is both the Bird-Carreau and Carreau-Yasuda models exhibit the empirical observed Newtonian approximations at relatively high and low shear rates.

Fluid Dynamics Solutions

The choice of model ultimately depends on what is to be shown by any proceeding calculations. Where small arteries or veins are modelled with non-compliant walls [1] this is usually the velocity field in order to identify sluggish and/or as-certain recirculating (often mistaken for turbulence) flow, in addition to high shearing affects suitably quantified by the aforementioned SSR.

Figure 3 shows results from a series of Newtonian and non-Newtonian computations of the SSR at the wall conducted using the ANSYS-CFX commercial CFD code [1], compared with standard results from Newtonian-Poiseuille theory (i.e. $\gamma(R_o) = 2V_{max}/R_o$ [11]). Both analytic and CFD models are representative of a 15 mm long cylinders of 2.5 mm diameter with non-compliant walls [5,4]. The CFD models consisted of 229,813 cells, corresponding to just over 350,000 DOF with 25 inflation layers arranged in a logarithmic fashion from the outer wall of the vessel to a radius of 0.05 mm. The number of inflation layers being a reflection of mesh independence. Boundary conditions were applied in order to mimic the vessel in-situ; no slip outer wall, central symmetry and Poiseuille velocity profile at the inlet, i.e. $V_{in}(r) = V_{max} \{1 - (r/R_o)^2\}$, with $V_{max} = 0.7$ obtained from Doppler ultrasonography measurements of the deep inferior epigastric arteries in pre-operative breast reconstruction patients [5]. To reduce computational expense, an absolute pressure of zero was applied to the outlet as the calculations were shown to be unaffected when compared to the more realistic aforementioned Poiseuille velocity profile. The latter being clearly demonstrated by the two CFD Newtonian calculations indicated in figure 3. Values of the SSR were interpolated from 20 equally spaced points from a line located on the circumference of the vessel parallel and at a distance R_o ($= 1.25\text{mm}$) from the symmetry plane.

The Newtonian CFD model predicts lower values of the SSR when compared to the Newtonian-Poiseuille analytic model counterpart; this is due to the under-prediction of the maximum

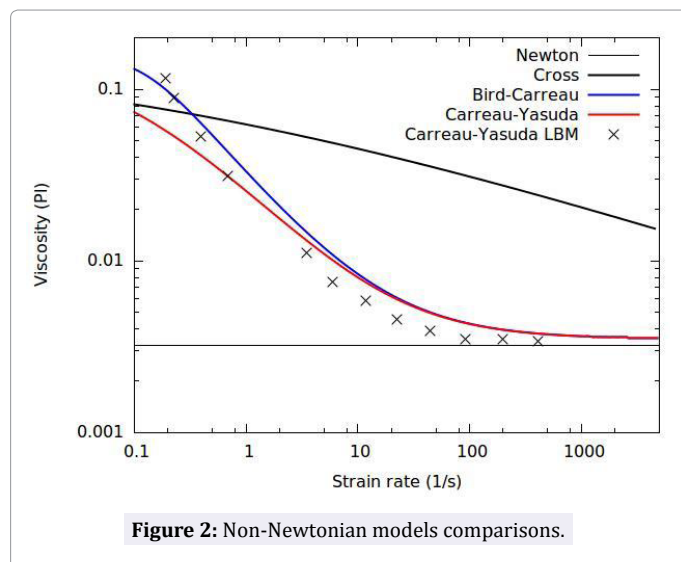


Figure 2: Non-Newtonian models comparisons.

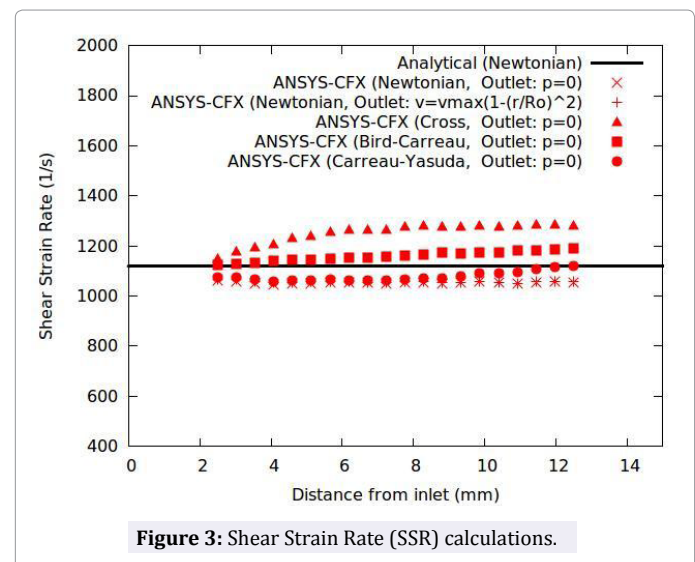


Figure 3: Shear Strain Rate (SSR) calculations.

velocity in a semi-inviscid core outside the inflation layers of the CFD models. Lower predicted values when compared with the Newtonian case are also evident from CFD computations via the Carreau-Yasuda model. On the other hand, higher values of the SSR compared to those calculated via the Newtonian model are predicted by the less realistic Cross and Bird-Carreau models. Most striking from data presented in figure 3 is, neglecting edge effects; all of the data remain practically constant for the vast majority of the vessel wall. Hence, each of the models predicts similar values of SSR due to each of them approximating a Newtonian case, albeit with slightly different effective viscosity. This can be explained with reference back to figure 2 since the value of the SSR is greater than 1000 per second where all the three of the curves and the LBM [10] data effectively converge.

Closure

Given the obvious assumptions and rudimentary models discussed throughout this note, it would appear to validate Prof. Steinman's bold statement [2]. The addition of the caveat "when the SSR of the particular flow exceeds 1000 per second, blood can indeed be modelled as a Newtonian rather than non-Newtonian fluid" is appropriate in this case. Furthermore, for the small diameter blood vessels of interest to our ongoing research [5,4], slight variations in the viscosity are apparent depending on the type of shear-thinning model used as indicated in table 1.

No inference from the models described here can be made with reference to the calibre of vessel at which non-Newtonian behavior becomes important; clearly further work should be carried out to this end. Importantly, the Newtonian and Carreau-Yasuda CFD modelling procedure described here consistently underestimate the peak velocity when compared with classical Newtonian-Poiseuille theory. Whence, the Carreau-Yasuda is employed in vessels of this size, a tendency toward non-Newtonian behavior is naturally favored, due to a consequential calculation of a lower

| Model | Viscosity (μ) |
|----------------|---------------------|
| Newtonian | 3.719 ± 0.036 |
| Cross | 3.129 ± 0.289 |
| Bird-Carreau | 3.381 ± 0.164 |
| Carreau-Yasuda | 3.636 ± 0.296 |
| Average | 3.499 ± 0.677 |

Table 1: Apparent viscosity values. Values calculated from the CFD data at the 99 % confidence level.

SSR. This said, analytical models show that the Carreau-Yasuda renders almost identical results to that of the Newtonian case, clearly indicating a redundancy with respect to its inclusion when modelling 2.5 mm diameter blood vessels, with the flow-rates used herein [5]. This note has therefore demonstrated that at small blood vessel diameters, the difference between a Newtonian and non-Newtonian solver is negligible. The results would of course change with viscosity, but not enough at these flow rates to make a non-Newtonian solver necessary. It is also pointed out that CFD cannot simulate blood with total accuracy, but it is a tool that can be incredibly valuable both for research and in clinical applications. Indeed, CFD is already being used in some centers to plan patient-specific procedures for vascular diseases [8].

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